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MAXIMUM DAY TEMPERATURES AND THE TEPHIGRAM ($t\phi$ DIAGRAM)

By E. GOLD, F.R.S.

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MAXIMUM DAY TEMPERATURES AND THE TEPHIGRAM
($t\,\phi$ Diagram)

By E. GOLD, F.R.S.

[*August* 30, 1932]

When the upper air temperatures from an early morning aeroplane ascent are plotted on a tephigram, the lower part of the curve is usually isothermal or inversional in character. An interesting problem, and one of practical importance to the forecaster, is to determine how much the temperature at the surface is likely to rise during the day owing to the effect of solar radiation. The problem is of practical importance—principally—in quiet weather, when the path of the air in the course of the day will be substantially over the land. The problem is also simplified, as well as of more importance, when there is little or no cloud. The following note has been written with a view to assisting the interpretation of the tephigram in connexion with this problem.

The strict theoretical determination of the amount by which the solar radiation can raise the temperature involves not only the incoming solar radiation, but also the amount reflected (including the amount scattered upwards) and the net increase of outward radiation of long wave length from the earth and lower layers of the atmosphere. (Much of the increased radiation from the earth's surface is almost immediately absorbed by the lowest few hundred feet of the atmosphere, and is therefore effective in warming these layers.) It involves, also, the heat required to warm the earth's surface; and, in addition, the amount of evaporation from the earth's surface; because the heat required to effect this evaporation has to be subtracted from the heat available for warming the atmosphere. It involves, further, a consideration of the amount of energy used in lifting the centre of gravity of the column of warmed air.

An *upper* limit can, however, be set for the increase of temperature by neglecting all these subsidiary quantities, and assuming that the whole of the solar radiation received by the atmosphere in the course of the day is available for raising the temperature of the lowest layers. This upper limit will, naturally, be well above the actual limit. Experience shows, for example, that the maximum temperature on a fine day in the summer is usually reached by about 3 p.m.; so that at least 10 per cent of the incoming solar radiation, i.e. the radiation which comes after 3 p.m., could be ignored, because it plays no part in raising the temperature but only plays a part in maintaining it when it has been raised.

The tephigram.—For convenience of reference a small section of a tephigram is shown in Fig. I. The framework of the diagram is shown by the thin and pecked lines. The lines sloping down from left to right are isobars and the pressure in millibars is written against these lines. The thin vertical lines are isotherms. These two

sets of lines really form the co-ordinate system, and simultaneous observations of pressure and temperature are plotted according to them.

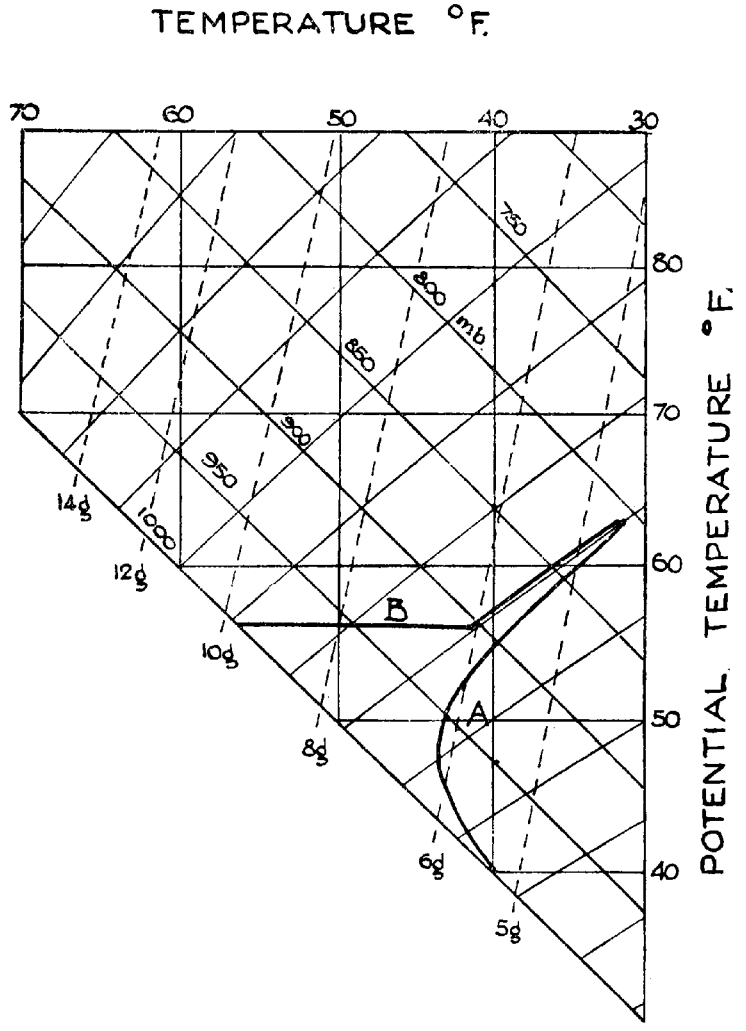

FIG. 1.—Section of $t\phi$ diagram.

Besides these two sets of lines, there are shown on the diagram :—

(*a*) Thin (straight) horizontal lines. These are lines of equal potential temperature, or dry adiabatics.

(*b*) Thin (not straight) lines sloping up from left to right. These are saturated adiabatics. If a mass of air saturated with water vapour is allowed to expand adiabatically, the corresponding values of pressure and temperature when plotted will lie along, or parallel to, these lines.

(*c*) Pecked lines sloping upwards from left to right. These lines show the number of grams which a kilogram of air would contain if it were saturated with water vapour, and had the pressure and temperature corresponding with a point on one of these lines.

The thick curve marked A on the diagram represents the results of a hypothetical set of observations of corresponding pressures and temperatures at different heights. The corresponding humidities are not shown ; it is assumed on this occasion that the air at an altitude of about 1,000 feet is practically saturated and, therefore, contains about 6 grams of water vapour per kilogram of air. The state of the atmosphere represented by the curve A is very stable in the very lowest layers, and comparatively stable at higher levels.

The curve B on the diagram represents the result which would be obtained if the lower layers of air in the condition represented by A were warmed until the atmosphere was in a convective state up to the level where the two curves A and B meet.

The effect on the temperature-pressure curve of the warming of the surface due to solar radiation will be, in the first place, to make this curve approximately a dry adiabatic in the very lowest layers. As the day advances the dry adiabatic curve will extend to lower pressures, i.e. to greater heights, until, if the process is continued long enough, it reaches the line of water-vapour content determined from the aeroplane's observations of the dry-bulb and wet-bulb thermometers at a height of about 1,000 feet. (We take the value at this height because it is more likely to be representative of the general water-vapour content of the lower layers than the corresponding value determined from the observations at the surface. In practice there is not—usually—a great difference between the two, but the value at the surface may either be too low, owing to the deposition of dew *from* the surface layer, or it may be too high, when the ground is wet owing to evaporation *into* the surface layer.)

After the dry adiabatic has reached this line of water-vapour content, the temperature-pressure curve B will proceed along the saturated adiabatic, until it reaches the actual curve A derived from the aeroplane ascent. The area between these two curves :—

A, the curve of actual observations made in the morning, and

B, the adiabatic curve through the value of surface temperature,

represents—approximately—the amount of energy required to effect the change. Note that B is a curve which progressively changes its position as the surface temperature rises, while A is the fixed curve based on early morning observations.

If, then, we determine what area S of the tephigram corresponds with the day's solar radiation, the area between the curves A and B must not exceed the area S.

The area between the curves A and B increases as the surface temperature increases, so that the condition just mentioned sets a limit to the value of the surface temperature.

Energy corresponding with 1 cm.^{2} on the tephigram.—On the tephigram used in the Forecast Division of the Meteorological Office, the distance measured along an isotherm between the isobaric lines for 900 and 1,000 mb. is approximately 30 mm. The distance between two isotherms differing by 1° F. is exactly 2 mm.

Thus an area of 60 mm.^{2} on the diagram represents, approximately, the energy required to raise the temperature of the layer of air between 1,000 and 900 mb. by 1° F. Actually, if heat were applied to this layer of air until its temperature had risen by 1° F., its centre of gravity would also have risen by approximately 1 metre, and the energy required to effect this lifting of the centre of gravity would be derived from the heat supplied.

The heat required by each column of air standing on a base of area 1cm.^{2} in order to raise the temperature of the layer of air by 1° F., if we assume the specific heat of air to be ·24, is approximately $\frac{5}{9} \times 24$ gm. cal. $= 13\frac{1}{3}$ gm. cal.

The energy required to lift the centre of gravity of the column of air through 1 metre is approximately 10^7 ergs $= \frac{1}{4}$ gm. cal.

Thus the total amount of energy represented by the area mentioned, 60 mm.^{2}, is approximately $13\frac{1}{2}$ gm. cal. per cm.^{2}

Thus an area of 1 cm.^{2} on the tephigram corresponds with a supply of energy equal to 22·5 gm. cal. per cm.^{2}

The area on the $t\ \phi$ diagram of the squares formed by the isotherms and dry adiabatics is approximately 4 cm.^{2}; this area, therefore, corresponds with a supply of energy equal to about 90 gm. cal. per cm.^{2}. A glance at the tephigram shows that if the lower layer of the atmosphere between pressures of 1,000 and 950 mb. were originally isothermal, it would require 45 gm. cal. per cm.^{2} to turn it into an adiabatic condition, and the effect of this would be to raise the surface temperature by approximately 10° F.

The area included between two isobars and two isotherms is not quite constant; the isobars get further apart as the pressure decreases. But for the levels and for the degree of accuracy with which we are concerned the area may be taken as constant.

Daily amounts of solar radiation and heights at which they would change an isothermal state to an adiabatic state.—The solar radiation entering the atmosphere in the course of a day varies during the year; in June it is more than five times what it is in December. Table 1[1] shows in column (2) the approximate amount in gm. cal. per cm.^{2} received *in one day* near the middle of each month.

[1] The values have been derived from the values given for lat. 50° N. in Shaw's "Manual of Meteorology," Vol. II, p. 4. To put Shaw's units into gm. cal./cm.^{2} multiply them by ·86.

TABLE I

(1)	(2) Daily amount	(3) Area on tephigram	(4) No. of large squares on tephigram	(5) Area for 50 per cent only	(6) Thickness of layer (Isothermal to adiabatic)	(7)	(8) Increase of surface temperature
	gm. cal./cm.^{2}	cm.^{2}		cm.^{2}	mb.	ft.	° F.
Jan. ..	210	9	2	4½	95	3000	15
Feb. ..	350	16	4	8	126	4000	20
Mar. ..	530	24	6	12	155	5000	25
April ..	730	33	8	16	182	5700	28
May ..	920	41	10	20	202	6400	32
June ..	980	44	11	22	210	6600	33
July ..	950	42	10½	21	205	6500	32
Aug. ..	820	39	10	20	198	6200	31
Sept. ..	620	28	7	14	167	5300	26
Oct. ..	410	18	5	9	134	4200	21
Nov. ..	250	11	3	5½	105	3300	16
Dec. ..	175	8	2	4	90	2800	14

The third column of the table gives the number of cm.^{2} on the tephigram corresponding with the values in the second column. The fourth column gives the corresponding number of squares on the tephigram. It is easy to deduce from this column, and the tephigram, the height to which solar radiation would turn an *isothermal* atmosphere into an adiabatic atmosphere in a single day, if the whole of the solar radiation were available for warming the lowest layers of the atmosphere. In the middle of December, if the surface pressure were 1,000 mb., the adiabatic state would be extended up to approximately 875 mb.; in March it would extend to just below 800 mb.; in June and July it would extend to about 750 mb. The corresponding increases in the surface temperature would be approximately 20° F. in December, 35° F. in March, and 45° F. in June and July.

These values are all upper limits, well above the limits which could actually be reached. In the first place, the amount of solar radiation reaching the earth and layers of the atmosphere in immediate proximity to the earth is not much more than 60 per cent of the amount reaching the outside of the atmosphere, even in the middle of the day. Then there is the additional outward radiation to space owing to the increased temperature of the earth's surface, and the reflection of incident solar radiation back to space: the latter may be taken roughly as 20 per cent of the incident radiation; for snow it would be much more than 20 per cent, for water appreciably less. Against these, which subtract from the energy available for warming the surface layer, must be set the increased indirect radiation from the sky, which adds to the energy available. We should still be on the safe side if we took the actual amount of solar energy available on a clear day for warming the lower layers up to the time of maximum temperature, as *not more than* 50 per cent of the total amount received in the upper layers of the atmosphere.

The number of cm.^{2} corresponding with the assumption of 50 per cent is given in column (5). Note that this is still, normally, an upper limit.,

It is not difficult to show that the thickness of the layer which would be changed from an isothermal to an adiabatic state (dry adiabatic) by the addition at the base of the layer of heat represented by $n$cm.^{2} on the tephigram is approximately $\sqrt{2000n}$ expressed in millibars of pressure. The corresponding value expressed in feet is approximately $1000\sqrt{2n}$.

Columns (6) and (7) of the table give the values derived from these two formulæ by using for n the values of column (5). The corresponding increase in the value of the surface temperature is approximately $5\sqrt{2n}$ in °F. The values derived from this formula are given in column (8).

Corrected amount of solar radiation available for warming the lower layers.—We may get a closer estimate of the energy available for raising the temperature of the lower layers by making allowances for the radiation from the earth's surface, the incoming diffuse radiation of short wave length, the reflected radiation, and the evaporation of water from the surface.

As an allowance for the radiation from the earth's surface, we take 30 per cent of the total radiation, as this is approximately the part of the radiation which will not be absorbed in the lowest layers of the atmosphere. The part of the radiation from the earth's surface which is absorbed by the lower layers of the atmosphere is available for warming them; most of it is required for *maintaining* their temperature, and only the excess is effective in *raising* their temperature. There will not, in the course of a day, be much increased radiation from the lower atmospheric layers to space or to the upper atmosphere in consequence of the increase in temperature of the lower layers; for example when the temperature has been raised in the layers to a height of 1,000 feet, most of the radiation for these layers will be absorbed in the next layer up to 2,000 feet. Accordingly, no allowance is made for increased radiation upwards from the lower layers.

The values of the non-absorbed part for the period from 1 hour after sunrise up to about 3 p.m., for each month of the year, are shown in column (4) of Table II. (The approximate number of hours is shown in column (2).) The temperatures in column (3) are the temperatures assumed for the radiating surface on a clear day. They make no pretence at exactness, but they are probably not far from the truth. In any case a variation of 10° F. will not substantially modify the results. The diffuse short-wave radiation for the period in question is taken, also as a rough approximation, from the paper by W. H. and L. H. G. Dines (*London, Mem. R. Meteor. Soc.* **2**, No. 11, 1927). The values are shown in column (5). The reflected solar radiation is taken to be 20 per cent of the incident radiation, the latter being assumed to be 50 per cent of the total solar radiation. The values are shown in

TABLE II—CORRECTED AMOUNTS OF SOLAR RADIATION AVAILABLE, AFTER DEDUCTIONS AND ADDITIONS IN RESPECT OF TRANSMITTED, DIFFUSE AND REFLECTED RADIATION AND EVAPORATION

(1)	(2)	(3)	(4)	(5) Radiation.	(6)	(7) Evaporation.	(8) Evaporation.	(9)	(10)
	Hours	Assumed temp.	30 per cent of full	Diffuse short-wave	Allowance of 20 per cent reflected	Estimated amount	Heat required	Total allowance	Resulting available energy
		° F.	gm. cal./cm.²	gm. cal./cm.²		mm.	gm. cal./cm.²	gm. cal./cm.²	gm. cal./cm.²
Jan. ..	6	45	50	25	20	0·3	20	65	40
Feb. ..	7	50	70	30	35	0·5	30	105	70
Mar. ..	8	60	90	35	50	1·0	60	165	100
April ..	9	80	110	45	70	1·5	90	225	140
May ..	10	100	130	55	90	2·0	120	285	175
June ..	10	110	150	55	95	2·0	120	310	180
July ..	10	110	150	55	95	2·0	120	310	165
Aug. ..	9	110	135	45	80	1·5	90	260	150
Sept. ..	8	90	110	35	60	1·0	60	195	115
Oct. ..	7½	70	85	30	40	0·5	30	125	80
Nov. ..	6½	50	65	25	25	0·3	20	85	40
Dec. ..	6	45	50	20	17	0·2	10	57	30

column (6). The estimated evaporation on a clear day, expressed in mm. of water, is shown in column (7), and the heat required to effect it is shown in column (8). In estimating the evaporation I have taken the table in Vol. II of Shaw's "Manual of Meteorology" as a guide. The total allowance obtained by adding columns (4), (6) and (8) and subtracting column (5) is given in column (9). The final net amount of radiation available for warming, obtained by subtracting column (9) from 50 per cent of the values in column (2) of Table I, is shown in column (10).

The thickness of the layer which would be changed from an isothermal to an adiabatic state by these reduced amounts, and the corresponding increase in surface temperature are given in Table III.

TABLE III—CORRECTED HEIGHTS TO WHICH AN ISOTHERMAL STATE WOULD BE CHANGED TO AN ADIABATIC STATE BY SOLAR RADIATION IN ONE DAY

(1)	(2) Thickness	(3) Thickness	(4) Increase of surface temp.	(5) Number of squares
	mb.	ft.	° F.	
Jan.	60	1900	9	0·4
Feb.	80	2500	12	0·7
Mar.	95	3000	15	1·1
April	110	3500	17	1·5
May	120	3900	19	1·9
June	125	4000	20	2·0
July	120	3800	19	1·8
Aug.	110	3600	18	1·7
Sept.	100	3200	16	1·3
Oct.	85	2700	13	0·9
Nov.	60	1900	9	0·7
Dec.	50	1600	8	0·4

The fifth column gives the number of squares on the tephigram equivalent to the area which would be included between the initial and final pressure-temperature curves. It appears that a single day's heating will not be likely to cover an area greater than two squares, even in the middle of summer.

The values in this fifth column are the critical values of the area S which are intended for practical application to the tephigram.

Usually the atmosphere is not isothermal in the early morning: on calm clear mornings there is an inversion in the very lowest layers, so that the temperature at 1,000 feet is well above the surface temperature. Account must be taken of this in using the values of column (4); they cannot be directly compared with the daily range of surface air temperature.

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